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Analysis of a Stochastic Inventory Model with Exponential Demand using Fuzzy Optimization and Intuitionistic Fuzzy Optimization Techniques

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ABSTRACT: In this paper a fixed reorder quantity system with lost sales is discussed. An item wise Multi Objective Stochastic Inventory model[MOSIM] is analyzed here. Considering the exponential demand the model is illustrated numerically.

KEYWORDS: Multi Objective Stochastic Inventory model, Fixed Reorder Quantity system, Intuitionistic Fuzzy Optimization

I. INTRODUCTION

Fuzzy mathematical programming has been applied to several fields like project network, reliability optimization, transportation, media selection for advertising; air pollution regulation etc. problems formulated in fuzzy environments. Detail literature on fuzzy linear and non-linear programming with application is available in two well-known books of Lie and Hwang (1992, 1994). Walter (1992) discussed the single period inventory problem with uniform demand. In inventory problem, fuzzy set theory has not been much used. Park (1987) examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with cost data. Roy and Maiti (1995, 1998) solved the classical EOQ models in fuzzy environment with fuzzy objective goal and constraint by fuzzy non-linear programming and fuzzy additive goal programming techniques. As stated in Karsak and Tolga (2001) the market demand, quality of products can neither be assessed by crisp values nor random processes. So, linguistic variables or fuzzy numbers can be used. H.C. Chang (2004) developed EOQ model having imperfect quality without shortages in which demand and defective rate are taken as triangular fuzzy numbers. On the other hand, fuzzy set theory has been widely developed and various modifications and generalizations have appeared. One of them is the concept of intuitionistic fuzzy (IF) sets. They consider not only the degree of membership to a given set, but also the degree of rejection such that the sum of both values is less than 1. Applying this concept it is possible to reformulate the optimization problem by using degrees of rejection of constraints and values of the objective that are non-admissible. The degrees of acceptance and of rejection can be arbitrary (the sum of both have to be less than or equal to 1). Bellman and Zadeh (1970) first introduced fuzzy set theory in decision-making processes. Later, Tanaka, et-al. (1974) considered the objectives as fuzzy goals over the \bot -cuts of a fuzzy constraint set and Zimmermann (1976) showed that the classical algorithms could be used to solve a fuzzy linear programming problem. Intuitionistic Fuzzy Set (IFS) was introduced by K. Atanassov (1986) and seems to be applicable to real world problems. Atanossov also analyzed Intuitionistic fuzzy sets in a more explicit way. Atanassov(1989) discussed an Open problems in intuitionistic fuzzy sets theory. An Interval valued intuitionistic fuzzy sets was analyzed by Atanassov and Gargov(1999). Atanassov and Kreinovich(1999) implemented Intuitionistic fuzzy interpretation of interval data. The temporal intuitionistic fuzzy sets are discussed also by Atanossov[1999]. In the present paper after discussion of the fixed rerder system with lost sales numerically it is established that IFO technique is better method than FO technique to solve this Multi Objective Stochastic model.



II. MATHEMATICAL MODEL

2.1 Fixed Reorder Quantity System with Lost Sales

Here we consider the fixed reorder quantity policy when all shortages are lost. Here the policy is to order a lot size Q when the inventory level drops to a reorder point r and it is supposed that the inventory position of an item is monitored after every transaction. The demand in any given interval of time is a random variable and the expected value of demand in a unit of time, say a year, is D. We let x denote the demand during the lead-time and f (x) denote its probability distribution.

We make the same assumptions as in the backorders case, except that the shortage cost π now includes the lost profit of an item. The procurement cost per cycle is A+CQ.

The shortages cost per cycle is $\pi \overline{b}(r)$, where $\overline{b}(r)$ is the expected number of shortages per cycle and is a function of reorder point r. In the lost sales case, the net inventory and the on-hand inventory are the same. The on-hand inventory at the end of a cycle, immediately prior to a receipt of a lot, is a $(x, r) = \max(0, r - x)$, where x is the lead-time demand. The expected on-hand inventory at the end of a cycle is:

$$\overline{a}(r) = \int_0^\infty (r - x) f(x) dx$$
$$= \int_0^\infty (r - x) f(x) dx + \int_r^\infty (x - r) f(x) dx$$
$$= r - \mu + \overline{b}(r)$$

The average annual cost is:

$$K(Q,r) = \frac{AD}{Q} + CD + h(\frac{Q}{2} + r - \mu) + (h + \frac{\pi D}{Q})\overline{b}(r)$$

Model I: Single Objective Stochastic Inventory Model [SOSIM]

The following model may be considered:

Minimize average annual cost under floor space constraints. It is a Single Objective Stochastic Inventory Model [SOSIM]

The model can be stated as:

$$\operatorname{Min} K(Q_1, Q_2, \dots, Q_n, r_1, r_2, \dots, r_n) = \sum_{i=1}^n \left(\frac{A_i D_i}{Q_i} + C_i D_i + h_i \left(\frac{Q_i}{2} + r_i - \mu_i \right) + \left(h_i + \frac{\pi_i D_i}{Q_i} \right) \overline{b_i}(r_i) \right)$$

$$\dots (2.1)$$

subject to the constraints

$$\sum_{i=1}^{n} p_i Q_i \le F$$

$$Q_i \ge 0 \ (i = 1, 2, ..., n)$$

Model II: Multi Objective Stochastic Inventory Model [MOSIM] To solve the problem (3.5) as a MOSIM, it can be reformulated as:

$$Min \ K_i(Q_i, r_i) = \left(\frac{A_i D_i}{Q_i} + C_i D_i + h_i \left(\frac{Q_i}{2} + r_i - \mu_i\right) + \left(h_i + \frac{\pi_i D_i}{Q_i}\right)\overline{b_i}(r_i)\right) \qquad \dots (2.2)$$

subject to the constraints

$$p_i Q_i \le F$$
 (i = 1, 2,, n).
 $Q_i \ge 0$ (i = 1, 2,, n).





2.2 Stochastic Mode: Demand Follows Exponential distribution

We assume that demand for the period for the ith item is a random variable, which follows exponential distribution. Then the probability density function $f_i(x)$ are given by:

$$f_i(\mathbf{x}) = \lambda_i e^{(-\lambda_i \mathbf{x})} , \quad \mathbf{x} > 0 \quad \text{for } \mathbf{i} = 1, 2, ..., \mathbf{n}.$$

= 0 , otherwise
So, $\overline{b}_i(r_i) = \frac{e^{-\lambda_i r_i}}{(-\lambda_i)} \quad \text{for } \mathbf{i} = 1, 2, ..., \mathbf{n}$

Where, $\overline{b}_i(r_i)$ are the expected number of shortages per cycle and all these values of $\overline{b}_i(r_i)$ affects the model of fixed reorder quantity system with lost sales.

III. MATHEMATICAL ANALYSIS

3.1 Multi-Objective Non-Linear Programming Problem [MONLP]

A Multi-Objective Non-Linear Programming (MONLP) or Vector Minimization problem (VMP) may be taken in the following form:

 $\begin{array}{l} \text{Min } f(x) = [f_1(x), f_2(x), f_3(x), \dots, f_k(x)]^T \\ \text{Subject to } x \in X = \{x \in \mathbb{R}^n : g_j(x) \leq \text{or} = \text{ or } \geq b_j \text{ for } j = 1, \dots, m \} \\ \text{and} \qquad l_i \leq x_i \leq u_i \ (i = 1, 2, \dots, n). \end{array}$

Zimmermann (1978) showed that fuzzy programming technique could be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used:

STEP 1: Solve the MONLP (3.1) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

STEP 2: From the result of step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

Here x^1, x^2, \ldots, x^k are the ideal solutions of the objective functions $f_1(x), f_2(x), \ldots, f_k(x)$ respectively.

So
$$U_r = \max \{f_r(x^1), f_r(x^2), \dots, f_r(x^k)\}$$

and $L_r = f_r^*(x^r)$.

 $[L_r \text{ and } U_r \text{ be lower and upper bounds of the } r^{th} \text{ objective functions } f_r(x) \text{ for } r = 1, 2, \dots, k].$

STEP 3: Using aspiration level of each objective of the MONLP (3.1) may be written as follows: Find x so as to satisfy

 $\begin{array}{ll} f_r(x) \stackrel{\scriptstyle <}{\leq} L_r & (\mbox{ for } r=1,2,\ldots,k) \\ x \ \epsilon \ X \end{array}$

Here objective functions of (3.1) are considered as fuzzy constraints. These types of fuzzy constraints can be quantified by eliciting a corresponding membership function:





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....(3.2)

....(3.3)

$$\begin{split} \mu_r \left[f_r(x) \right] &= \ 0 \ or \to 0 \ \ if \ f_r(x) \geq U_r \\ &= \ d_r(x) \qquad if \ L_r \leq f_r(x) \leq U_r \ (r = 1, \, 2, \, \dots, \, k) \\ &= \ 1 \ or \to 1 \ \ if \ \ f_r(x) \leq L_r \end{split}$$

Here $d_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$ (r=1, 2, ..., k). Having elicited the membership functions (as in (3.2)) μ_r [$f_r(x)$] for r = 1, 2,, k, introduce a general aggregation function

$$\mu_{\bar{D}}(x) = \mu_{\bar{D}}(\mu_1(f_1(x)), \mu_2(f_2(x)), \dots, \mu_k(f_k(x))).$$

So a fuzzy multi-objective decision making problem can be defined as

Max $\mu_{\bar{D}}(x)$

subject to $x \in X$.

Here we adopt the type of fuzzy decision as:

Fuzzy decision based on minimum operator (like Zimmermann's approach (1976)). In this case (3.2) is known as $FNLP_M$.

(According to min-operator)

Max $\mu_j f_j(x)$ Subject to $\mu_r f_r(x) \ge \mu_j f_j(x)$ $x \in X, \quad 0 \le \mu_r f_r(x) \le 1$, for r, j = 1, 2,, k; r \ne j.(3.4)

STEP 4: Solve (3.4) to get optimal solution.

3.2 Formulation of Intuitionistic Fuzzy Optimization [IFO]

When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other, then IF sets can be used as a more general tool for describing uncertainty.

To maximize the degree of acceptance of IF objectives and constraints and to minimize the degree of rejection of IF objectives and constraints, we can write:

 $\max \mu_{i}(\overline{X}), \overline{X} \in \mathbb{R}, i = 1, 2, \dots, K + n$ $\min \upsilon_{i}(\overline{X}), \overline{X} \in \mathbb{R}, i = 1, 2, \dots, K + n$ Subject to $\upsilon_{i}(\overline{X}) \ge 0,$ $\mu_{i}(\overline{X}) \ge \upsilon_{i}(\overline{X})$ $\mu_{i}(\overline{X}) + \upsilon_{i}(\overline{X}) < 1$ $\overline{X} \ge 0$

Where $\mu_i(\overline{X})$ denotes the degree of membership function of (\overline{X}) to the i^{th} IF sets and $\nu_i(\overline{X})$ denotes the degree of non-membership (rejection) of (\overline{X}) from the i^{th} IF sets.

3.3 An Intuitionistic Fuzzy Approach for Solving MOIP with Linear Membership and Non-Membership Functions

To define the membership function of MOIM problem, let L_k^{acc} and U_k^{acc} be the lower and upper bounds of the k^{th} objective function. These values are determined as follows: Calculate the individual minimum value of each objective

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function as a single objective IP subject to the given set of constraints. Let $\overline{X}_1^*, \overline{X}_2^*, \dots, \overline{X}_k^*$ be the respective optimal solution for the k different objective and evaluate each objective function at all these k optimal solution. It is assumed here that at least two of these solutions are different for which the kth objective function has different bounded values. For each objective, find lower bound (minimum value) L_k^{acc} and the upper bound (maximum value) U_k^{acc} . But in intuitionistic fuzzy optimization (IFO), the degree of rejection (non-membership) and degree of acceptance (membership) are considered so that the sum of both values is less than one. To define membership function of MOIM problem, let L_k^{rej} and U_k^{rej} be the lower and upper bound of the objective function $Z_k(\overline{X})$ where L_k^{acc} $\leq {L_k}^{rej} \leq {U_k}^{rej} \leq {U_k}^{acc}$. These values are defined as follows:

The linear membership function for the objective $Z_k(\overline{X})$ is defined as:

$$\mu_{k}(Z_{k}(\overline{X})) = \begin{cases} 1 & \text{if } Z_{k}(\overline{X}) \leq L_{k}^{acc} \\ \frac{U_{k}^{acc} - Z_{k}(\overline{X})}{U_{k}^{acc} - L_{k}^{acc}} & \text{if } L_{k}^{acc} \leq Z_{k}(\overline{X}) \leq U_{k}^{acc} \\ 0 & \text{if } Z_{k}(\overline{X}) \geq U_{k}^{acc} \end{cases} \qquad \dots (3.5)$$

$$v_{k}(Z_{k}(\overline{X})) = \begin{cases} 1 & \text{if } Z_{k}(\overline{X}) \geq U_{k}^{rej} \\ \frac{Z_{k}(\overline{X}) - L_{k}^{rej}}{U_{k}^{rej} - L_{k}^{rej}} & \text{if } L_{k}^{rej} \leq Z_{k}(\overline{X}) \leq U_{k}^{rej} \\ 0 & \text{if } Z_{k}(\overline{X}) \leq L_{k}^{rej} \end{cases} \qquad \dots (3.6)$$

Lemma: In case of minimization problem, the lower bound for non-membership function (rejection)) is always greater than that of the membership function (acceptance).

Now, we take new lower and upper bound for the non-membership function as follows:

K

$$L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ where } 0 < t < 1$$
$$U_k^{rej} = U_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ for } t = 0$$

Following the fuzzy decision of Bellman-Zadeh (1970) together with linear membership function and non-membership functions of (3.5) and (3.6), an intuitionistic fuzzy optimization model of MOIM problem can be written as: max μ (\overline{X}) $\overline{X} \in \mathbb{R}$ k = 1.2

min
$$\mu_k(\overline{X}), \overline{X} \in \mathbb{R}, k = 1, 2, \dots, K$$

Subject to
 $\upsilon_k(\overline{X}) \ge 0,$
 $\mu_k(\overline{X}) \ge \upsilon_k(\overline{X})$
 $\mu_k(\overline{X}) \ge \upsilon_k(\overline{X})$
 $\mu_k(\overline{X}) + \upsilon_k(\overline{X}) < 1$
 $\overline{X} \ge 0$
The problem of equation (3.7) can be reduced following Angelov (1997) to the following form:
 $Max \ \alpha - \beta$
 $\dots (3.8)$





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Subject to

 $Z_k(\overline{X}) \leq U_k^{acc} - \alpha (U_k^{acc} - L_k^{acc})$ $Z_{k}(\overline{X}) \leq L_{k}^{rej} + \beta(U_{k}^{rej} - L_{k}^{rej})$ $\beta \ge 0$ $\alpha \geq \beta$ $\alpha + \beta < 1$ $\overline{X} > 0$

Then the solution of the MOIM problem is summarized in the following steps:

Step 1. Pick the first objective function and solve it as a single objective IP subject to the constraint, continue the process K-times for K different objective functions. If all the solutions (i.e. $\overline{X}_1^* = \overline{X}_2^* = \dots = \overline{X}_k^*$ $(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ same, then one of them is the optimal compromise solution and go to step 6. Otherwise go to step 2. However, this rarely happens due to the conflicting objective functions.

Then the intuitionistic fuzzy goals take the form

 $Z_k(\overline{X}) \cong L_k(\overline{X})^*_k \ k = 1, 2, \dots, K.$

Step 2. To build membership function, goals and tolerances should be determined at first. Using the ideal solutions, obtained in step 1, we find the values of all the objective functions at each ideal solution and construct pay off matrix as follows:

 $Z_1(\overline{X}_1^*)$ $Z_2(\overline{X}_1^*)$ $Z_k(\overline{X}_1^*)$ $Z_1(\overline{X}_2^*)$ $Z_2(\overline{X}_2^*)$ $Z_k(\overline{X}_2^*)$ $\begin{bmatrix} Z_{1}(\overline{X}_{k}^{*}) & Z_{2}(\overline{X}_{k}^{*}) & \dots & \dots & Z_{k}(\overline{X}_{k}^{*}) \end{bmatrix}$

Step 3. From Step 2, we find the upper and lower bounds of each objective for the degree of acceptance and rejection corresponding to the set of solutions as follows:

$$U_k^{acc} = \max(Z_k(\overline{X}_r^*)) \quad \text{and} \ L_k^{acc} = \min(Z_k(\overline{X}_r^*))$$

$$1 \le r \le k \qquad 1 \le r \le k$$
For linear membership functions

For linear membership func

 $L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ where } 0 < t < 1$

$$U_k^{reg} = U_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ for } t = 0$$

Step 4. Construct the fuzzy programming problem of equation (3.7) and find its equivalent LP problem of equation (3.8).

Step 5. Solve equation (3.8) by using appropriate mathematical programming algorithm to get an optimal solution and evaluate the K objective functions at these optimal compromise solutions Step 6. STOP.



IV. ILLUSTRATION OF MOSIM OF A FIXED REORDER QUANTITY SYSTEM WITH LOST SALES, WHERE DEMAND FOLLOWS EXPONENTIAL DISTRIBUTION

A fixed reorder quantity system with lost sales[MOSIM] (2.2) with exponential demand is considered with the following data:

 $p_1 = 3 \text{ ft}^2, p_2 = 3 \text{ ft}^2, F = 50000 \text{ ft}^2, A_1 = \$70, A_2 = \$80, h_1 = \$1, h_2 = \$1.5, D_1 = 5000, D_2 = 4000, C_1 = \$5, C_1 = \$7, \pi_1 = \$2.2, \pi_1 = \$2, \lambda_1 = 0.2, \lambda_2 = 0.5.$

We illustrate numerically the Multi Objective Stochastic Inventory Model[MOSIM] with Fuzzy Optimization [FO] and Intuitionistic Fuzzy Optimization[IFO] Techniques.

TABLE 1

Method	Q1*	$\mathbf{Q_2}^*$	\mathbf{r}_1^*	r 2 [*]	K ₁ [*] (\$)	K2 *(\$)
FO	901.28	772.26	7.01	0.98	24989.11	56673.33
IFO	831.77	932.82	11.22	4.87	24943.12	56637.23

V. CONCLUSION

In this paper when Multi Objective Stochastic Inventory Model[MOSIM] is analyzed using Fuzzy Optimization [FO] and Intuitionistic Fuzzy Optimization[IFO] Techniques it is observed that Average annual costs K_1 and K_2 are more minimized in case of IFO than FO technique.

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