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Acceleration Due to Gravity

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ABSTRACT: Acceleration due to gravity is the acceleration gained by an object due to gravitational force. Its SI unit is m/s^2. It has both magnitude and direction; hence, it's a vector quantity. Acceleration due to gravity is represented by g. The standard value of g on the surface of the earth at sea level is 9.8 m/s^2.

Acceleration Due to Gravity – Formula, Unit and Values

Acceleration Due to Gravity (g)	
Symbol	g
Dimensional Formula	$M^0L^1T^{-2}$
SI Unit	ms^{-2}
Formula	$g = GM/r^2$
Values of g in SI	9.806 ms^{-2}
Values of g in CGS	980 cm s^{-2}

KEYWORDS- Acceleration Due to Gravity, gravitational force, surface of the earth, magnitude and direction

I.INTRODUCTION

Gravity is the force with which the earth attracts a body towards its centre. Let us consider two bodies of masses, m_a and m_b. Under the application of equal forces on two bodies, the force in terms of mass is given by

$m_b = m_a [a_A/a_B]$; this is called an inertial mass of a body.

Under the gravitational influence on two bodies,[1,2]

- $F_A = GMm_A/r^2$,
- $F_B = GMm_B/r^2$,[3]
- $m_B = [F_B/F_A] \times m_A$

The above mass is called a gravitational mass of a body. According to the principle of equivalence, the inertial mass and gravitational mass are identical. We will be using this while deriving acceleration due to the gravity given below.Suppose a body [test mass (m)] is dropped from a height 'h' above the surface of the earth [source mass (M)]; it begins to move downwards with an increase in velocity as it reaches close to the earth's surface.We know that the velocity of an object changes only under the action of a force; in this case, the force is provided by gravity.Under the action of gravitational force, the body begins to accelerate toward the earth's centre, which is at a distance 'r' from the test mass.[5,7,8]

- Then, $ma = GMm/r^2$ (Applying principle of equivalence)
- $\Rightarrow a = GM/r^2$ . . . . (1)
- The above acceleration is due to the gravitational pull of the earth, so we call it acceleration due to gravity; it does not depend upon the test mass. Its value near the surface of the earth is 9.8 ms^{-2}.
- Therefore, the acceleration due to gravity (g) is given by = GM/r^2.

Formula of Acceleration Due to Gravity

Force acting on a body due to gravity is given by f = mg[10,11,12]

Where f is the force acting on the body, g is the acceleration due to gravity, and m is the mass of the body.

According to the universal law of gravitation, $f = GmM/(r+h)^2$

Where,

- f = Force between two bodies
- G = Universal gravitational constant (6.67×10^{-11} Nm^2/kg^2)
- m = Mass of the object
- M = Mass of the earth
- r = Radius of the earth
- h = Height at which the body is from the surface of the earth

As the height (h) is negligibly small compared to the radius of the earth, we re-frame the equation as follows:

$f = GmM/r^2$

Now, equating both expressions,

$mg = GmM/r^2$

$\Rightarrow g = GM/r^2$

Therefore, the formula of acceleration due to gravity is given by $g = GM/r^2$

Note: It depends on the mass and radius of the earth.

This helps us understand the following:

- All bodies experience the same acceleration due to gravity, irrespective of their mass.
- Its value on earth depends upon the mass of the earth and not the mass of the object.[13,15]

II.DISCUSSION

Acceleration Due to Gravity on the Surface of Earth

Earth is assumed to be a uniform solid sphere with a mean density. We know that,

Density = mass/volume

Then, $\rho = M/[4/3\ \pi R^3]$

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$\Rightarrow M = \rho \times [4/3\ \pi R^3]$

We know that $g = GM/R^2$.

On substituting the values of M, we get,

$g = 4/3\ [\pi\rho RG]$

At any distance ‘r’ from the centre of the earth,

$g = 4/3\ [\pi\rho RG]$

The value of acceleration due to gravity ‘g’ is affected by

- Altitude above the earth’s surface.
- Depth below the earth’s surface.
- The shape of the earth.
- Rotational motion of the earth.

Variation of g with Height

Acceleration due to gravity at a height (h) from the surface of the earth

Consider a test mass (m) at a height (h) from the surface of the earth. Now, the force acting on the test mass due to gravity is

$F = GMm/(R+h)^2$

Where M is the mass of the earth, and R is the radius of the earth. The acceleration due to gravity at a certain height is ‘h’, then[1,8,10]

$mg_h = GMm/(R+h)^2$

$\Rightarrow g_h = GM/[R^2(1+ h/R)^2\ ]$ . . . . . . (2)

The acceleration due to gravity on the surface of the earth is given by

$g = GM/R^2$ . . . . (3)

On dividing equations (3) and (2), we get

$g_h = g\ (1+h/R)^{-2}$. . . . . . (4)

This is the acceleration due to gravity at a height above the surface of the earth. Observing the above formula, we can say that the value of g decreases with an increase in the height of an object, and the value of g becomes zero at an infinite distance from the earth.[19,20,21]

⇒ Check: Kepler’s Laws of Planetary Motion

Approximation Formula:

From equation (4)

when h << R, the value of g at height ‘h’ is given by $g_h = g/(1 – 2h/R)$

Variation of g with Depth

Consider a test mass (m) taken to a distance (d) below the earth's surface, the acceleration due to gravity at that point (g_d) is obtained by taking the value of g in terms of density.

On the surface of the earth, the value of g is given by

$g = 4/3 \times \pi\rho RG$

At a distance (d) below the earth's surface, the acceleration due to gravity is given by

$g_d = 4/3 \times \pi\rho \times (R - d) G$

On dividing the above equations, we get,

$g_d = g (R - d)/R$

- When the depth d = 0, the value of g on the surface of the earth $g_d = g$.
- When the depth d = R, the value of g at the centre of the earth $g_d = 0$.

Variation of g Due to the Shape of the Earth

As the earth is an oblate spheroid, its radius near the equator is more than its radius near the poles. Since for a source mass, the acceleration due to gravity is inversely proportional to the square of the radius of the earth, it varies with latitude due to the shape of the earth.

$g_p/g_e = R^2_e/R^2_p$

Where g_e and g_p are the accelerations due to gravity at the equator and poles, R_e and R_p are the radii of the earth near the equator and poles.[22,23,25]

From the above equation, it is clear that acceleration due to gravity is more at the poles and less at the equator. So if a person moves from the equator to the poles, their weight decreases as the value of g decreases.

Variation of g Due to Rotation of Earth

Consider a test mass (m) is on a latitude making an angle with the equator. As we have studied, when a body is under rotation, every particle in the body makes circular motions about the axis of rotation. In the present case, the earth is under rotation with a constant angular velocity ω, then the test mass moves in a circular path of radius 'r' with an angular velocity ω.

This is the case of a non-inertial frame of reference, so there exists a centrifugal force on the test mass ($mr\omega^2$). Gravity is acting on the test mass towards the centre of the earth (mg).

As both these forces are acting from the same point, these are known as co-initial forces, and as they lie along the same plane, they are termed co-planar forces.

We know from the parallelogram law of vectors if two co-planar vectors are forming two sides of a parallelogram, then the resultant of those two vectors will always be along the diagonal of the parallelogram.

Applying the parallelogram law of vectors, we get the magnitude of the apparent value of the gravitational force at the latitude.

$(mg')^2 = (mg)^2 + (mr\omega^2)^2 + 2(mg)(mr\omega^2) \cos(180 - \theta)$ . . . . . . (1)

We know 'r' is the radius of the circular path and 'R' is the radius of the earth, then $r = R\cos\theta$.

Substituting $r = R \cos\theta$. we get,

$g' = g - R\omega^2\cos^2\theta$

Where g′ is the apparent value of acceleration due to gravity at the latitude due to the rotation of the earth, and g is the true value of gravity at the latitude without considering the rotation of the earth.

At poles, $\theta = 90° \Rightarrow g' = g$.

At the equator, $\theta = 0° \Rightarrow g' = g - R\omega^2$.

III.RESULTS

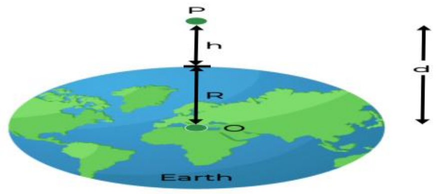
Acceleration due to gravity (or acceleration of gravity) or gravity acceleration is the acceleration caused by the gravitational force of attraction of large bodies. As we know that the term acceleration is defined as the rate of change of velocity with respect to a given time. Scientists like Sir Isaac Newton and Lord Henry Cavendish soon discovered that this increase in speed, or acceleration, was caused by a different force known as gravity by studying objects falling to the Earth in a variety of circumstances. According to definitions, gravity is a force that pulls objects toward the center of mass, like the Earth. Conversely, acceleration describes how an object’s velocity or speed changes over time. Hence, the value of acceleration due to gravity is 9.8 m/s^2 on earth.

Factor affecting Acceleration due to Gravity

- Shape of Earth: It is known that the shape of the earth is not spherical it’s quite oval so the gravitational force is different at different places. The force of attraction is maximum at the pole of the earth approximately 9.82 m/s^2 as the radius of the earth is minimum at the pole. While the force of gravitation is minimum at the equator of the earth at approximately 9.78 m/s^2 as the radius of the earth is maximum at the equator
- Altitude: When a body moves away from the surface of the earth the force of attraction decreases as the distance between the earth and the body increases.
- Depth: When a body is put inside the earth’s surface the acceleration due to gravity becomes less.

Effects on g due to Height (h)

Consider an object (of mass m) P at a height h from the surface of the earth, R be the radius of the earth as shown in the figure below:



Effect on g due to Height h

The gravitational force F acting on the mass m is,

$F = GMm/(R+h)^2$

where, M is the mass of the Earth.

Since, $F = mg_h$, where g_h is the acceleration due to gravity at height h. Then the above equation becomes:

$mg_h = GMm/(R+h)^2$

$g_h = GM/r^2\,(1+h/r)^2$

$= (GM/r^2)/(1+h/r)^2$

Since, $g = GM/r^2$

Therefore,

$g_h = g/(1+h/r)^2$

Now, if h is much lesser than the radius of the earth, the value of g at height h is given by:

$g_h = g/(1 - 2h/r)$

Effects on g due to Depth D

Consider an object (of mass m) P at a depth d from the surface of the earth, R be the radius of the earth as shown in the figure below:

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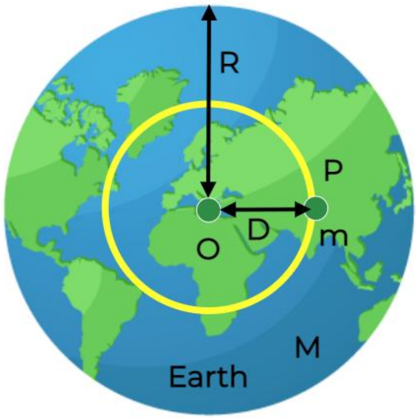


Effect on g due to Depth D

The acceleration due to gravity at the surface of Earth in terms of density is:

$g = 4/3 \times \pi\rho \times RG$
At depth D,

$g_D = 4/3 \times \pi\rho \times (R-D)G$
On dividing both equations we get,

$g_d = g \times \pi\rho \times (R-D)$
Now two cases can be possible:

Case 1: If depth D is equal to the radius of the earth i.e. D = R, then:
$g_d = 0$
Case 2: If depth D = 0, i.e. the object is at the surface of earth, then
$g_d = g$

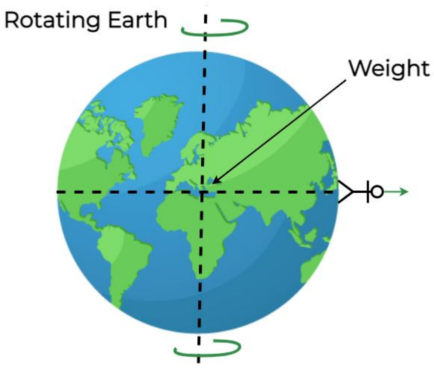
Effects on g due to Shape of Earth
The radius of the earth, an oblate spheroid, is greater towards the equator than it is farther from the poles. The acceleration caused by gravity changes with latitude due to the shape of the earth since it is inversely proportional to the square of the earth's radius for a given mass[12,17,19]

where,

- R_e and R_p are the radii of Earth at the equator and the poles, and
- g_e and g_p are the acceleration due to gravity at the equator and poles, respectively.

Hence, from the above-mentioned equation, it is obvious that the equator experiences less, and the poles have more gravitational acceleration. Therefore, when g lowers, a person's weight falls as they move from the equator to the poles.

Effects on g due to the Rotation
The variation in g is due to the centrifugal force acting on the rotation of the earth. When the earth is rotating, all the objects tend to experience a centrifugal force that won't act in the direction of gravity.



Effect on g due to Rotation of Earth

IV.CONCLUSIONS

Acceleration due to gravity, acceleration of gravity or gravity acceleration may refer to:

- Gravitational acceleration, the acceleration caused by the gravitational attraction of massive bodies in general
- Gravity of Earth, the acceleration caused by the combination of gravitational attraction and centrifugal force of the Earth
- Standard gravity, or g, the standard value of gravitational acceleration at sea level on Earth[25]

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