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Partial Differential Equation- A Mathematical Enumeration Article

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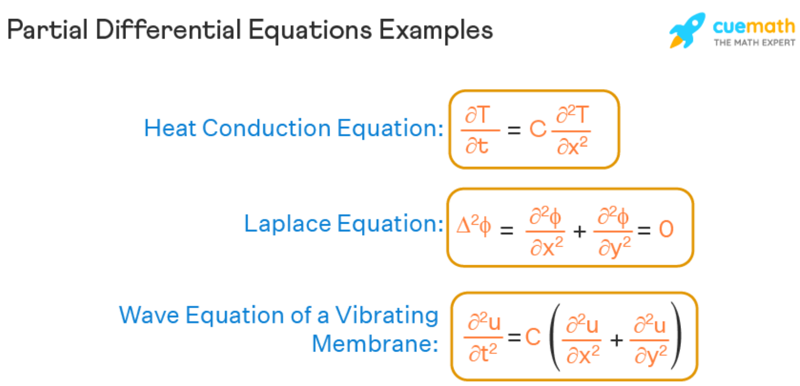
ABSTRACT: Partial differential equations are equations that consist of a function with multiple unknown variables and their partial derivatives. In other words, partial differential equations help to relate a function containing several variables to their partial derivatives. These equations fall under the category of differential equations.Partial differential equations are very useful in studying various phenomena that occur in nature such as sound, heat, fluid flow, and waves.

KEYWORDS: partial differential equations, function, phenomena, variables, function

I.INTRODUCTION

Partial differential equations are abbreviated as PDE. These equations are used to represent problems that consist of an unknown function with several variables, both dependent and independent, as well as the partial derivatives of this function with respect to the independent variables.[15] Partial differential equations can be defined as a class of differential equations that introduce relations between the various partial derivatives of an unknown multivariable function. Such a multivariable function can consist of several dependent and independent variables. An equation that can solve a given partial differential equation is known as a partial solution.[1]

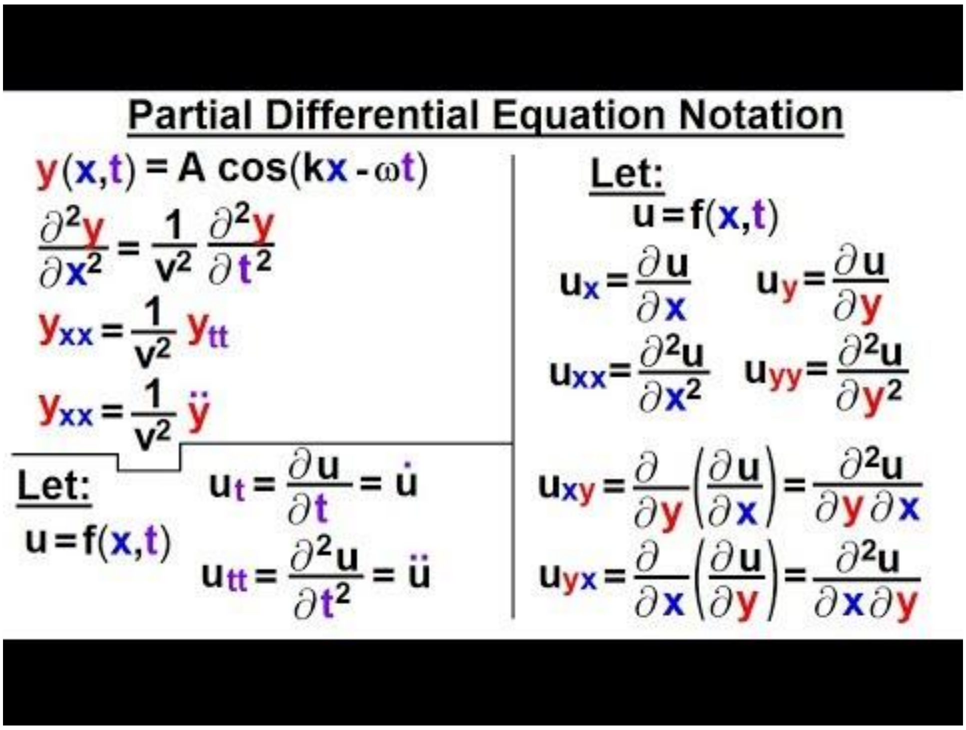
Partial Differential Equations Example



Partial differential equations can prove to be difficult to solve. Hence, there are certain techniques such as the separation method[2], change of variables, etc. that can be used to get a solution to these equations. Order and degree of partial differential equations are used to categorize partial differential equations.[16] The most commonly used partial differential equations are of the first-order and the second-order. Partial differential equations can be broadly divided into 4 types based on the order of the partial derivatives as well as the nature of the equation[3]. These are given below:

First-Order Partial Differential Equations

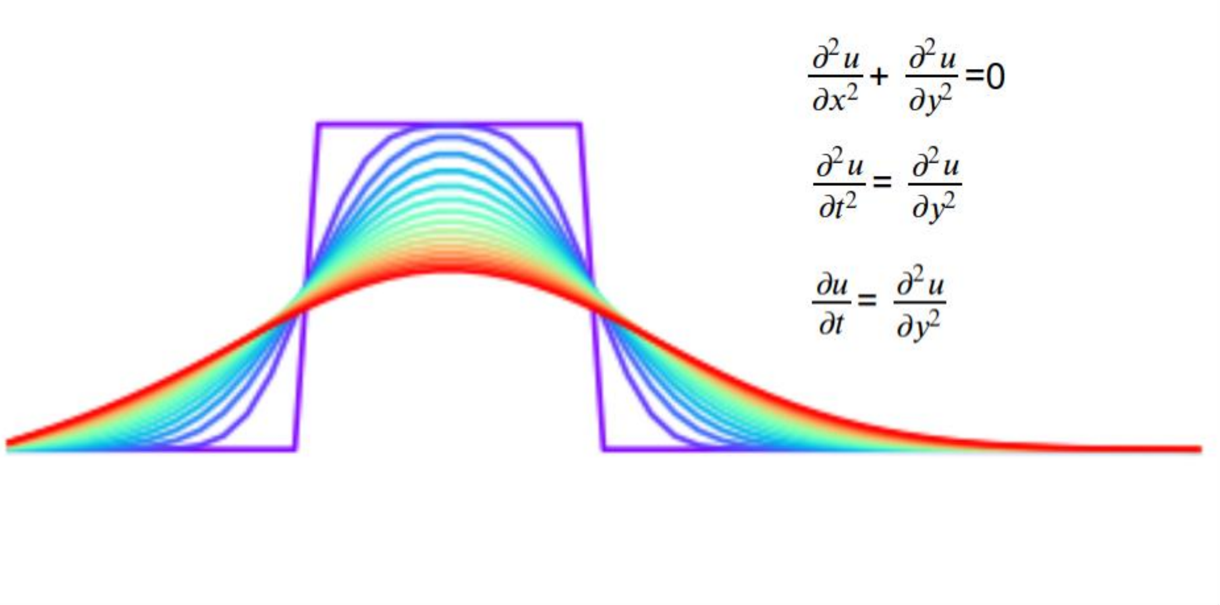
Partial differential equations where the highest partial derivatives of the unknown function are of the first order are known as first-order partial differential equations. If the equation has n number of variables then we can express a first-order partial differential equation as F (x1x1, x2x2, ...,xnxn, kx1kx1, ...kxnkxn). [4]First-order PDEs can be both linear and non-linear. A linear partial differential equation is one where the derivatives are neither squared nor multiplied.[17]



Second-Order Partial Differential Equations

Second-order partial differential equations are those where the highest partial derivatives are of the second order. Second-order PDEs can be linear, semi-linear, and non-linear. Linear second-order partial differential equations are easier to solve as compared to the non-linear and semi-linear second-order PDEs. The general formula for a second-order partial differential equation is given as auxx+buxy+cuyy+dux+euy+fu=g(x,y)auxx+buxy+cuyy+dux+euy+fu=g(x,y).[18]

Here, a, b, c, d, e, f, and g are either real-valued functions of x and/or y or they are real constants.[5]



Quasi Linear Partial Differential Equations

In quasilinear partial differential equations, the highest order of partial derivatives occurs, only as linear terms. First-order quasi-linear partial differential equations are widely used for the formulation of various problems in physics and engineering.[6]

Homogeneous Partial Differential Equations

A partial differential equation can be referred to as homogeneous or non-homogeneous depending on the nature of the variables in terms. The partial differential equation with all terms containing the dependent variable and its partial derivatives is called a non-homogeneous PDE or non-homogeneous otherwise.[7]

II.DISCUSSION

Partial Differential Equations Classification

Suppose we have a linear second-order PDE of the form Auxxuxx + 2Buxyuxy + Cuyyuyy + other lower-order terms = 0. Then the discriminant of such an equation will be given by B^2 - AC. Using this discriminant, second-order partial differential equations can be classified as follows:

- **Parabolic Partial Differential Equations:** If B^2 - AC = 0, it results in a parabolic partial differential equation. An example of a parabolic partial differential equation is the heat conduction equation.[8]
- **Hyperbolic Partial Differential Equations:** Such an equation is obtained when B^2 - AC > 0. The wave equation is an example of a hyperbolic partial differential equation as wave propagation can be described by such equations.
- **Elliptic Partial Differential Equations:** B^2 - AC < 0 are elliptic partial differential equations. The Laplace equation is an example of an elliptic partial differential equation.[19]

Classification	Canonical Form	Type	Example
b^2 - ac > 0	∂2u∂ξ∂η+...=0∂2u∂ξ∂η+...=0	Hyperbolic Partial Differential Equation	Wave propagation equation
b^2 - ac = 0	∂2u∂η2+...=0∂2u∂η2+...=0	Parabolic Partial Differential Equation	Heat conduction equation
b^2 - ac < 0	∂2u∂α2+∂2u∂β2+...=0∂2u∂α2+∂2u∂β2+...=0	Elliptic Partial Differential Equation	Laplace equation

Partial Differential Equations Applications

Partial differential equations are widely used in scientific fields such as physics and engineering. Some applications of partial differential equations are given below:

- Partial differential equations are used to model equations to describe heat propagation. The equation is given by uxxuxx = utut[9]
- Propagation of light and sound is given by the wave equation. This equation is a second-order partial differential equation and is given by uxxuxx - uyyuyy = 0.
- The Black-Scholes equation is another important second-order partial differential equation that is used to construct financial models.[20]

III.RESULTS

A partial differential equation is an equation consisting of an unknown multivariable function along with its partial derivatives.There a broadly 4 types of partial differential equations. These are first-order, second-order, quasi-linear partial differential equations, and homogeneous partial differential equations.Second-order partial differential equations can be classified into three types - parabolic, hyperbolic, and elliptic.[10]

Many physically important partial differential equations are second-order and linear. For example:

- $u_{xx} + u_{yy} = 0$ (two-dimensional Laplace equation)
- $u_{xx} = u_t$ (one-dimensional heat equation)
- $u_{xx} - u_{yy} = 0$ (one-dimensional wave equation)[11]

The behaviour of such an equation depends heavily on the coefficients a, b, and c of $au_{xx} + bu_{xy} + cu_{yy}$. They are called elliptic, parabolic, or hyperbolic equations according as $b^2 - 4ac < 0$, $b^2 - 4ac = 0$, or $b^2 - 4ac > 0$, respectively. Thus, the Laplace equation is elliptic, the heat equation is parabolic, and the wave equation is hyperbolic.[21]
A 1-D PDE includes a function u(x,t) that depends on time t and one spatial variable x. The MATLAB PDE solver `pdepe` solves systems of 1-D parabolic and elliptic PDEs of the form

$$u_x = \frac{\partial u}{\partial x}$$

$$u_{xx} = \frac{\partial^2 u}{\partial x^2}$$

$$u_{xy} = \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$$

Partial differential equations (PDEs) are the most common method by which we model physical problems in engineering. Finite element methods are one of many ways of solving PDEs. This handout reviews the basics of PDEs and discusses some of the classes of PDEs in brief. The contents are based on Partial Differential Equations in Mechanics volumes 1 and 2 by A.P.S. Selvadurai and Nonlinear Finite Elements of Continua and Structures by T. Belytschko, W.K. Liu, and B. Moran.[12]

We usually come across three-types of second-order PDEs in mechanics. These are classified as elliptic, hyperbolic, and parabolic.[22]

The equations of elasticity (without inertial terms) are elliptic PDEs. Hyperbolic PDEs describe wave propagation phenomena. The heat conduction equation is an example of a parabolic PDE.

Each type of PDE has certain characteristics that help determine if a particular finite element approach is appropriate to the problem being described by the PDE. Interestingly, just knowing the type of PDE can give us insight into how smooth the solution is, how fast information propagates, and the effect of initial and boundary conditions.[13]

- In hyperbolic PDEs, the smoothness of the solution depends on the smoothness of the initial and boundary conditions. For instance, if there is a jump in the data at the start or at the boundaries, then the jump will propagate as a discontinuity in the solution. If, in addition, the PDE is nonlinear,[12] then shocks may develop even though the initial conditions and the boundary conditions are smooth. In a system modeled with a hyperbolic PDE, information travels at a finite speed referred to as the wavespeed. Information is not transmitted until the wave arrives.[23]
- In contrast, the solutions of elliptic PDEs are always smooth, even if the initial and boundary conditions are rough (though there may be singularities at sharp corners). In addition, boundary data at any point affect the solution at all points in the domain.
- Parabolic PDEs are usually time dependent and represent the diffusion-like processes. Solutions are smooth in space but may possess singularities. However, information travels at infinite speed in a parabolic system.[14]

IV.CONCLUSIONS

The theory of partial differential equations (PDE) is important both in pure and applied mathematics. On the one hand they are used to mathematically formulate many phenomena from the natural sciences (electromagnetism,[13] Maxwell's equations) or social sciences (financial markets, Black-Scholes model). On the other hand since the pioneering work on surfaces and manifolds by Gauss and Riemann partial differential equations have been at the centre of many important developments on other areas of mathematics (geometry, Poincare-conjecture).[15]

Subject of the module are four significant partial differential equations (PDEs) which feature as basic components in many applications: The transport equation, the wave equation, the heat equation, and the Laplace equation.[14] We will discuss the qualitative behaviour of solutions and, thus, be able to classify the most important partial differential equations into elliptic, parabolic, and hyperbolic type.[16]

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