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Vorticity in Fluid Dynamics (Mathematics)

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ABSTRACT: In continuum mechanics, vorticity is a pseudovector field that describes the local spinning motion of a continuum near some point (the tendency of something to rotate[1]), as would be seen by an observer located at that point and traveling along with the flow. It is an important quantity in the dynamical theory of fluids and provides a convenient framework for understanding a variety of complex flow phenomena, such as the formation and motion of vortex rings.[2][3] In a mass of continuum that is rotating like a rigid body, the vorticity is twice the angular velocity vector of that rotation. This is the case, for example, in the central core of a Rankine vortex.[5]

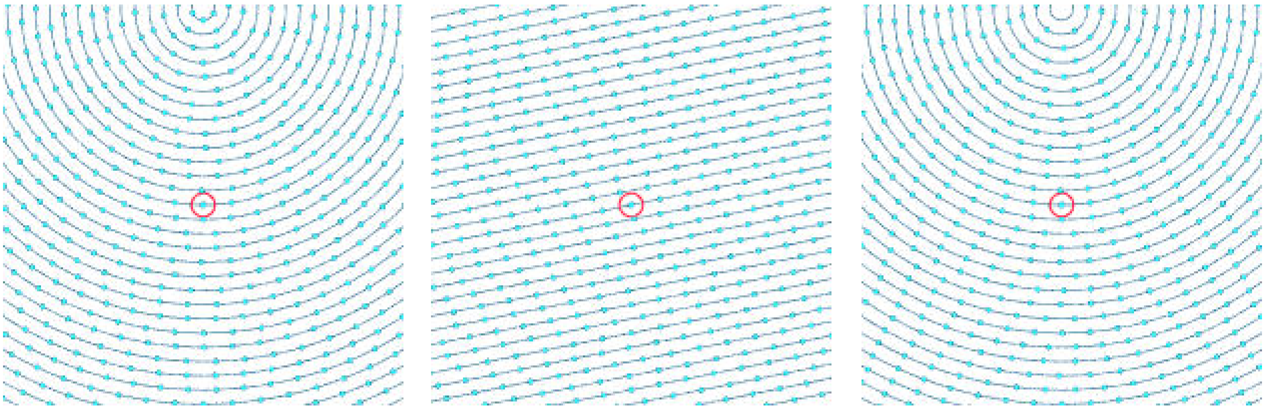
The vorticity may be nonzero even when all particles are flowing along straight and parallel pathlines, if there is shear (that is, if the flow speed varies across streamlines). For example, in the laminar flow within a pipe with constant cross section, all particles travel parallel to the axis of the pipe; but faster near that axis, and practically stationary next to the walls. The vorticity will be zero on the axis, and maximum near the walls, where the shear is largest.

KEYWORDS: vorticity, fluid dynamics, angular velocity, shear, laminar flow, continuum, rankine vortex

I.INTRODUCTION

Conversely, a flow may have zero vorticity even though its particles travel along curved trajectories. An example is the ideal irrotational vortex, [1,2]where most particles rotate about some straight axis, with speed inversely proportional to their distances to that axis. A small parcel of continuum that does not straddle the axis will be rotated in one sense but sheared in the opposite sense, in such a way that their mean angular velocity about their center of mass is zero.[3,4]

Example flows:



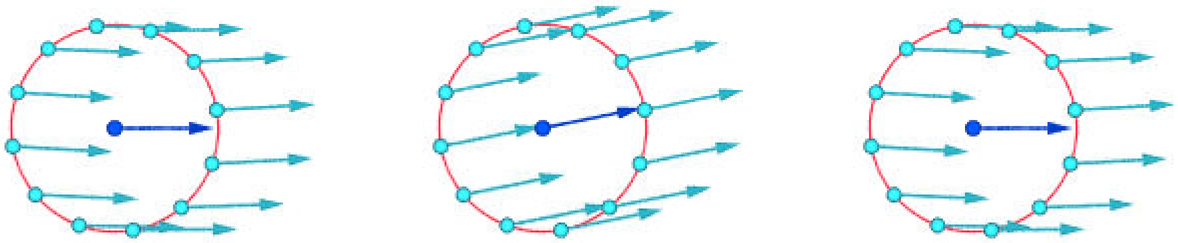
Rigid-body-like vortex
$v \propto r$

Parallel flow with shear

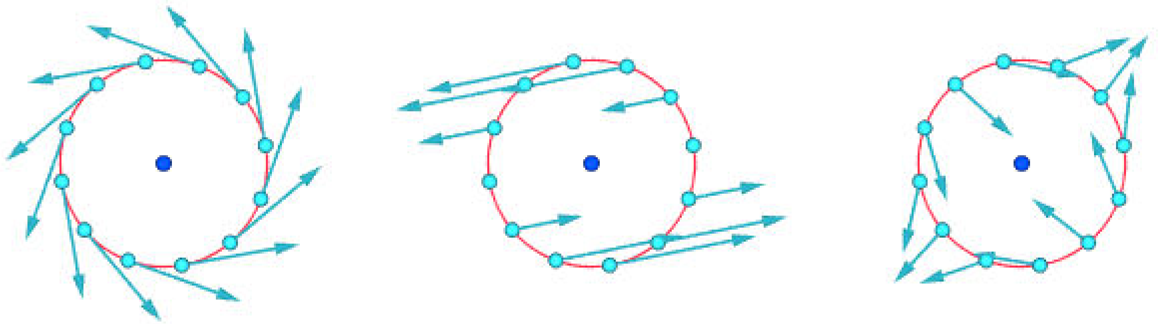
Irrotational vortex
$v \propto 1/r$

where v is the velocity of the flow, r is the distance to the center of the vortex and $\propto$ indicates proportionality.

Absolute velocities around the highlighted point:

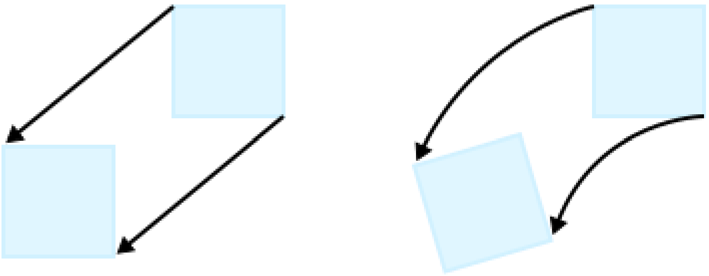


Relative velocities (magnified) around the highlighted point



Vorticity ≠ 0 Vorticity ≠ 0 Vorticity = 0

Another way to visualize vorticity is to imagine that, instantaneously, a tiny part of the continuum becomes solid and the rest of the flow disappears. If that tiny new solid particle is rotating, rather than just moving with the flow, then there is vorticity in the flow. In the figure below, the left subfigure demonstrates no vorticity, and the right subfigure demonstrates existence of vorticity.[5,6]



II.DISCUSSION

The evolution of the vorticity field in time is described by the vorticity equation, which can be derived from the Navier–Stokes equations.[7]

In many real flows where the viscosity can be neglected (more precisely, in flows with high Reynolds number), the vorticity field can be modeled by a collection of discrete vortices, the vorticity being negligible everywhere except in small regions of space surrounding the axes of the vortices. This is true in the case of two-dimensional potential flow (i.e. two-dimensional zero viscosity flow), in which case the flowfield can be modeled as a complex-valued field on the complex plane.[7,8]

Vorticity is useful for understanding how ideal potential flow solutions can be perturbed to model real flows. In general, the presence of viscosity causes a diffusion of vorticity away from the vortex cores into the general flow field; this flow is accounted for by a diffusion term in the vorticity transport equation.[8]

A vortex tube is the surface in the continuum formed by all vortex lines passing through a given (reducible) closed curve in the continuum. The 'strength' of a vortex tube (also called vortex flux)[10] is the integral of the vorticity across a cross-section of the tube, and is the same everywhere along the tube (because vorticity has zero divergence). It is a consequence of Helmholtz's theorems (or equivalently, of Kelvin's circulation theorem) that in an inviscid fluid the 'strength' of the vortex tube is also constant with time. Viscous effects introduce frictional losses and time dependence.[11]

In a three-dimensional flow, vorticity (as measured by the volume integral of the square of its magnitude) can be intensified when a vortex line is extended — a phenomenon known as vortex stretching.[12] This phenomenon occurs in the formation of a bathtub vortex in outflowing water, and the build-up of a tornado by rising air currents.[9,10]

A rotating-vane vorticity meter was invented by Russian hydraulic engineer A. Ya. Milovich (1874–1958). In 1913 he proposed a cork with four blades attached as a device qualitatively showing the magnitude of the vertical projection of the vorticity and demonstrated a motion-picture photography of the float's motion on the water surface in a model of a river bend.[13]

Rotating-vane vorticity meters are commonly shown in educational films on continuum mechanics (famous examples include the NCFMF's "Vorticity"[14] and "Fundamental Principles of Flow" by Iowa Institute of Hydraulic Research[15]).

In aerodynamics, the lift distribution over a finite wing may be approximated by assuming that each spanwise segment of the wing has a semi-infinite trailing vortex behind it. It is then possible to solve for the strength of the vortices using the criterion that there be no flow induced through the surface of the wing. This procedure is called the vortex panel method of computational fluid dynamics. The strengths of the vortices are then summed to find the total approximate circulation about the wing. According to the Kutta–Joukowski theorem, lift is the product of circulation, airspeed, and air density.[11,12]

III.RESULTS

The relative vorticity is the vorticity relative to the Earth induced by the air velocity field. This air velocity field is often modeled as a two-dimensional flow parallel to the ground, so that the relative vorticity vector is generally scalar rotation quantity perpendicular to the ground. Vorticity is positive when – looking down onto the earth's surface – the wind turns counterclockwise. In the northern hemisphere, positive vorticity is called cyclonic rotation, and negative vorticity is anticyclonic rotation; the nomenclature is reversed in the Southern Hemisphere.[13,14]

The absolute vorticity is computed from the air velocity relative to an inertial frame, and therefore includes a term due to the Earth's rotation, the Coriolis parameter.

The potential vorticity is absolute vorticity divided by the vertical spacing between levels of constant (potential) temperature (or entropy). The absolute vorticity of an air mass will change if the air mass is stretched (or compressed) in the vertical direction, but the potential vorticity is conserved in an adiabatic flow. As adiabatic flow predominates in the atmosphere, the potential vorticity is useful as an approximate tracer of air masses in the atmosphere over the timescale of a few days, particularly when viewed on levels of constant entropy.

The barotropic vorticity equation is the simplest way for forecasting the movement of Rossby waves (that is, the troughs and ridges of 500 hPa geopotential height) over a limited amount of time (a few days). In the 1950s, the first successful programs for numerical weather forecasting utilized that equation.

In modern numerical weather forecasting models and general circulation models (GCMs), vorticity may be one of the predicted variables, in which case the corresponding time-dependent equation is a prognostic equation.

In fluid mechanics, potential vorticity (PV) is a quantity which is proportional to the dot product of vorticity and stratification. This quantity, following a parcel of air or water, can only be changed by diabatic or frictional processes. It is a useful concept for understanding the generation of vorticity in cyclogenesis (the birth and development of a cyclone), especially along the polar front, and in analyzing flow in the ocean.[14]

Potential vorticity (PV) is seen as one of the important theoretical successes of modern meteorology. It is a simplified approach for understanding fluid motions in a rotating system such as the Earth's atmosphere and ocean. Its development traces back to the circulation theorem by Bjerknes in 1898,[1] which is a specialized form of Kelvin's circulation theorem. Starting from Hoskins et al., 1985,[2] PV has been more commonly used in operational weather diagnosis such as tracing dynamics of air parcels and inverting for the full flow field. Even after detailed numerical weather forecasts on finer scales were made possible by increases in computational power, the PV view is still used in academia and routine weather forecasts, shedding light on the synoptic scale features for forecasters and researchers.[3]

Baroclinic instability requires the presence of a potential vorticity gradient along which waves amplify during cyclogenesis.[15]

IV.CONCLUSIONS

The Ertel PV conservation theorem, states that for a dry atmosphere, if an air parcel conserves its potential temperature, its potential vorticity is also conserved following its full three-dimensional motions. In other words, in adiabatic motion, air parcels conserve Ertel PV on an isentropic surface. Remarkably, this quantity can serve as a Lagrangian tracer that links the wind and temperature fields. Using the Ertel PV conservation theorem has led to various advances in understanding the general circulation. One of them was "tropopause folding" process described in Reed et al., (1950).[9] For the upper-troposphere and stratosphere, air parcels follow adiabatic movements during a synoptic period of time. In the extratropical region, isentropic surfaces in the stratosphere can penetrate into the tropopause, and thus air parcels can move between stratosphere and troposphere, although the strong gradient of PV near the tropopause usually prevents this motion. However, in frontal region near jet streaks, which is a concentrated region within a jet stream where the wind speeds are the strongest, the PV contour can extend substantially downward into the troposphere, which is similar to the isentropic surfaces. Therefore, stratospheric air can be advected, following both constant PV and isentropic surfaces, downwards deep into the troposphere. The use of PV maps was also proved to be accurate in distinguishing air parcels of recent stratospheric origin even under sub-synoptic-scale disturbances. [16]

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